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Design of Radiation Therapy By Using Linear Programming

A Project Submitted to the Department of Mathematics Sciences University of Diyala in Partial Fulfillment for the Degree of B.Sc in Mathematics

BY

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Dedication

- To the candle of our life's (our prophet mohammed).
- To my best friend farah who changed my life.
- To my father who exterminated old to give the causes of success.
- To my mother who provided us with her tenderness.
- To my dearest brothers and sisters for their limitless love.
- To the best of knowledge we lit up our knowledge we lit up our ways(our teachers).

To all of them dedicate our work ….

Acknowledgements

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I would like to dedicate my deep gratefulness to our supervisor Dr.Adawiya Ali Mahmood.

Last but not least, all thanks and gratefulness to our family for their helping and supporting us in the course of studying and currying out the research. We thank every person who helped us even by sweet words that help us to submit this research .

Abstract

Modelers combine methods and skills in mathematics and computation with the specialised knowledge of healthcare experts to arrive together at appropriate solution to problem in healthcare.

This paper attempts to lay out a mathematical model to design radiation therapy by linear programming. This model is solved by using the Big M method.

Keywords: Mathematical model, linear programming, radiation therapy, healthcare .

1.Introduction

Many application of science make use of models. the term 'model' is usually used for a structure that has been built with the purpose of exhibiting features and characteristics of some other objects. More often, in operational research, we will be concerned with abstract models. these models will usually be mathematical in that algebraic symbolism will be used to mirror the internal relationships in the object (often an organization) being modeled. the essential feature of a mathematical model in operational research is that it involves a set of mathematical relationship (such as equations, inequalities and logical dependencies) that correspond to some more down -to-earth relationships in the real world (such as technological relationships, physical laws and marketing constraints).

Generally speaking, engineers and scientists try to understand, develop, or optimize "systems".

Here, "system" refer to the object of interest, which can be a part of nature (such as a plant cell, an atom, a galagxy ect.) or an artificial technological system

Over time, mathematical modeling has proven to be a useful tool for indentifying the patterns of disease spread during epidemics by projecting plausible scenarios using the best available information and data.

Mathematical model simulate complex systems in relatively fast time without the enormous costs of laboratory experiments and the biological variations. particularly for oncology, such models can be calibrated using experimental or clinical data and competing hypotheses of tumor progression can be evaluated and treatment options thoroughly analyzed before clinical intervention . the application of mathematical in pharmacokinetics by using one or two compartment models is discussed.

The reminder of this paper is structured as follows. Section 2 presents the linear programming model. Provides a design of radiation therapy problem. Section 4 presents formulation as a linear programming model with solution of the model. Conclusions are given in section 5.
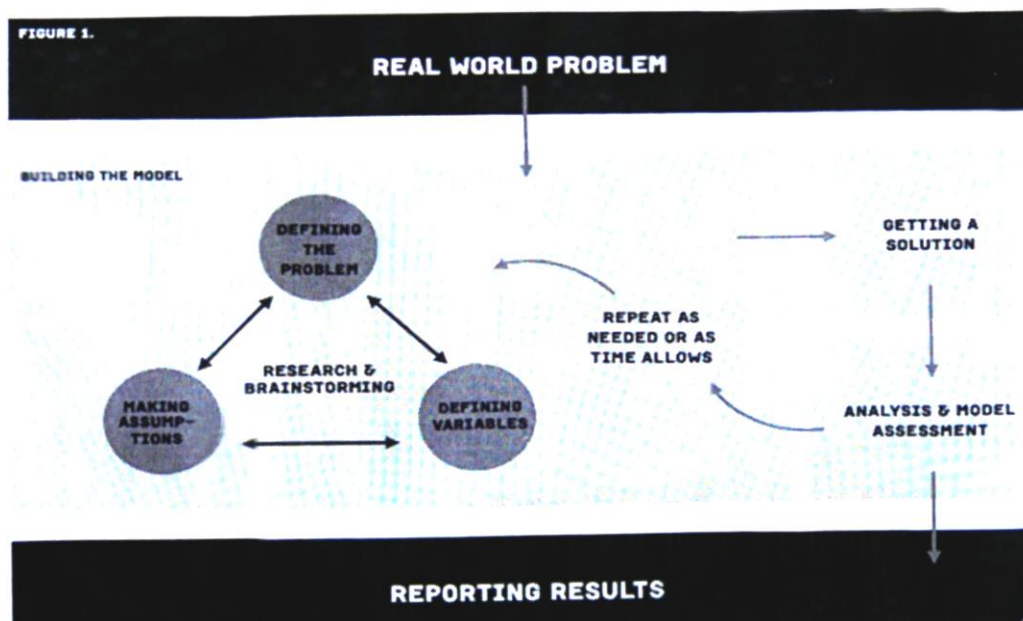
Chapter One

Building of mathematical model

1.1 Mathematical modeling

A mathematical model is representation of a system or scenario that is used to gain qualitative and or quantitative understanding of some real world problems and to predict future behavior. Models are used in a variety of disciplines ,such as biology, engineering , computer science, psychology ,sociology ,and marketing .Because models are abstractions of reality ,they can lead to scientific advances , provide the foundation for new discoveries , and help leaders make informed decision the aim of this section to help to demystify the process of how a mathematical model can be built . Building a useful math model dose not necessarily require advanced .

Mathematics or significant expertise in any of the fields listed above . It dose require a willingness to do some research , brainstorm , and jump right in and try something that may be out of your comfort zone .

Figure (1) will help to understand each of the components of math modeling . Its important to remember that this isnt necessarily a sequential list of steps ,math modeling is an iterative process, and the key steps may be revisted multiple times .Steps may be revested multiple times.

FIGURE 1.

REAL WORLD PROBLEM

BUILDING THE MODEL

DEFINING THE PROBLEM

GETTING A SOLUTION

REPEAT AS NEEDED OR AS TIME ALLOWS

RESEARCH & BRAINSTORMING

MAKING ASSUMP- TIONS

DEFINING VARIABLES

ANALYSIS & MODEL ASSESSMENT

REPORTING RESULTS

1.2 The problem statement

Modeling problem are often open ended. Some math modeling problems are clearly defined, while others are ambiguous. This means there is an opportunity for creative problem solving and interpretation. In some cases, it is up to the modeler to define the outputs of the model and which key concepts will be quantified. Defining the problem statement requires some research and brainstorming. The goal is a concise statement that explains what the model will predict.

We can summarize defining the problem statement in the following steps:

Step I: Often math modeling questions are worded in ways that allow for multiple approaches, so we should develop a concise restatment of the question at hand.

Step 2: Focus on subjective words that can be interpreted in different ways. Also, identify words that are not easily quantified. Examples include best, thrilling, efficient, robust and optimal.

Step 3: Explore the problem by doing a combination of research and brainstorming, keeping in mind our time constraints.

Step 4: Keep an open mind and a positive attitude: we can prune out ideas later that are not realistic.

Step 5: Brainstorming should be approached as if we had access to any data we need.

Step 6: visual diagrams, such as mind maps, can be a powerful tool leading to the structure of the model. Consider using the website free mind.

Step 7: In the end, we should have a concise statement that explains what the model will measure or predict.

1.3 Making assumptions

In presenting any scientific work to others, we need to explain how the results were achieved with explicit details so that they can be if we repeated explaining a chemistry experiment, for example, you need to list (among other things) which chemicals were used in what quantities and in what order. Other chemists would expect similar results only when they used the same chemicals and procedure. The list of assumptions for a model is as critical as the experimental procedure in performing a chemistry experiment. The assumptions tell the reader under what conditions the model is valid. Making assumptions can be one of the most intimidating parts of the modeling process for a novice, but it need not be! Assumptions are necessary and help we make a seemingly impossible question much more tractable. Many assumptions will follow quite naturally from the brainstorming process.

We can summarize making assumption in the following steps:

Step 1: Assumptions often come naturally from the process of brainstorming and defining the problem statement.

Step 2: we should do some preliminary research and may find data to help us make assumptions. In the absence of relevant data, make a reasonable assumption and justify the assumption in our write-up.

Step 3: Different assumptions can lead to different, equally valid models at different mathematical levels.

Step 4: *Not* all assumptions are made during the initial brainstorming some come as we move through the modeling process. Keep track of the assumptions we make and include a list of assumptions in our write-up of the model.

1.4 Definition variables

With the problem statement clearly defined and an initial set of assumptions made (a list that will likely get longer), we are ready to start to define the details of our model. Now is the time to pause to ask what a measure. Identifying these notions as variables, with units and some sense of their range is key to building the model. The purpose of a model is to predict or quantify something of interest. We refer to these predictions as the outputs of the model another term use for out puts are dependent variables. We will also have independent variables, or inputs to the model. Some quantities in a model might be held constant, in which case they are referred to as model parameters.

We can summarize definition variable in the following step:

Step 1: The problem statement should .determine the output of the model. The output will be dependent variables.

Step 2: The results of the initial brainstorming can provide insight into which variables should be independent variables and which should be fixed model parameters.

Step 3: Keep track of units because they can reveal relationships between variables.

Step 4: We will likely need to do some research and make additional assumptions to obtain values for necessary model parameters.

Step 5: Sub models or multiple models may be needed to generate some of the model input.

1.5 Building Solutions

Now that we have an initial mathematical model we will need to use that model to generate preliminary answers to the question at hand. The approach we take of course depends on the type of model we have and our background in mathematics. It may involve simply considering some different values of certain parameters to see how the output changes, it may involve techniques from calculus or differential equations, or it may involve using graphs to understand trends in data. In this chapter we will give we some strategies for choosing how to solve our problem. When we first approach any mathematical problem, we often look into our personal tool kit for a mathematical technique to use. Sometimes, if we start with the incorrect approach, a better approach will naturally emerge. So, the important thing is to just tackle it and see what happens!

We can summarize Building Solutions in the following steps:

step: How we build a solution may depend on what mathematical tools

Are available ours.

Step1: There is often more than one way to tackle a problem, so just start and see what happens.

Step3: If we don't immediately know how to solve the problem at hand ask ours the provided set of questions to help us get started.

Step, 4: Different solution methods can lead to solutions of different natures. This is perfectly acceptable.

1.6: Analysis and model assessment

Often we are so excited that we built and solved a mathematical model that we forget to step back and carefully examine the results. While this is understandable since it took hard work to get to that point, it is essential to ask ours, "Does my answer make sense?" Sometimes, the results indicate a mistake in the calculations. Other times may that we find additional or alternate assumptions are needed for the solution to be realistic. If the results do make sense, then further analysis is needed to assess the quality of the model. Recall that open-ended questions may have more than one solution and that the results depend on the assumptions made and the level of sophistication of the mathematics used. An honest evaluation is necessary to explain when the model is applicable and when it is not. In this section we will talk about ways to applicable us and how to assess the quality of your model. During process, we may gather some intuition about what the modeling process; we may gather some intuition about what the output will be like. Here we provide some pointers on how to answer the output will be question "Does my answer make sense?" by analyzing the output of the model.

Is the sign of the answer correct? For example, if our disease model is supposed to calculate the number of infected people at a certain time, then clearly an answer of -1000 would not make sense. Carefully check our calculations, especially if we are using software. For example, in Excel it is easy to select the wrong cell when defining a formula. Our model may be correct, but its implementation may be at fault.

. **Is the magnitude of the answer reasonable?** If we are trying to estimate the speed of a car, for example, then it wouldn't make sense it our model predicts a value of 1000 miles per hour. Sometimes, when the magnitude of a number is off, incorrect units may have been used somewhere in the process.

. **Does the model behave as expected?** If the output of your model is visualized with a graph or plot of any kind, then carefully look at the intercepts, the maximum or minimum values, or the long-term behavior. Were we expecting a horizontal asymptote, yet our graph just increases without bound? If we have a data set and believe there is a relationship between two variables, plot the data. A mathematical error in the sign of the slope will be immediately obvious. It could be that we had some assumptions which were neglected, erroneous units, unrealistic parameters, or that the software was used incorrectly.

. **Can you validate the model?** Sometimes it is possible to validate our model using available data. For example, if we used our roller coaster ranking model on the Top Thrill Dragster of Cedar Point, which held the record for the tallest roller coaster in the world and goes 120 mph, and the said it was only mediocre, then

likely our model is not doing what we want it to Now that we have verified that our model is correct, it is time to step back and consider the validity of our model. This includes identifying the strengths and weaknesses of our model and understanding at a deeper level the behavior of the model. Performing a sensitivity analysis, wherein we analyze how changes in the input and parameters impact the output, can contribute to understanding the behavior of our model Once we are convinced that the output is correct and the model is achieving what we want, assess the quality of model. This assessment needs be included in the write-up about our model to help people understand the conditions under which our model is applicable, which is strongly linked to the assumptions that were made along the way. It is necessary for us to provide an honest, exact assessment of the capabilities of our model. This is also a chance to highlight the strengths of the model. For example, even if a model was formulated using simple physics, it might require very little expert knowledge in order to provide meaningful insight. This can be a huge advantage over a more complex model that requires the user to program and run software, research other model parameters in order to fit the model to his or her own needs, or sort through complicated output to be able to draw a *conclusion.* It is also strength if the people who might use our model can understand it and have faith in it. If time allows, assessment and sensitivity analysis can lead to improvements in the model. Modeling, as pointed out earlier, is an iterative process, and refinements can almost always be made to develop more realistic model. If the modeling is being done in a timed setting such as for a competition or a homework assignment, then this may not be possible (although it is generally possible to indicate the type of refinements that would improve the model). However, for long-term projects, the model assessment really is an intermediate step before (possibly) starting the modeling loop over again. Discussing possible modifications to the model, even if we cannot make them, demonstrates that we are able to think beyond just the first approach.

We can <u>summarize analysis and model assessment in the following steps:</u>

Step l: Be sure to allocate time to analyze our results since it is indeed a critical part of the entire modeling process.

Step 2: Always examine the output we get from our model and ask ours if it makes sense. If our answer doesn't make sense, verify that we haven't made a mistake in implementing our model.

Step 3: If our solution is consistent with our assumptions but not consistent with the real-world phenomenon we are trying to describe, we may need to refine our model by adjusting our assumptions.

Step 4: List strengths and weaknesses of our model.

Step 5: Try to determine how sensitive our model is to parameters and assumptions.

Step 6: Include specific improvements we would have incorporated given more time.

Chapter two

Formulation of linear programming with some solution methods

2.1 Introduction to linear programming

Certain symbols are commonly used to denote the various components of a linear programming model. These symbols are listed below. A long with their interpretation for the general problem of allocating resources to activities

z=value of overall measure of performance

xi= level of activity j (for j=1,2,3... ,n)

c_j= increase in z that would result from each unit increase in level of activity j.

b_i= amount of resource i that available for allocation to activities (for i=1,2,...,m)

a_{ij}= amount of resource i consumed by each unit of activity j.

The model poses the problem in terms of making decisions about the levels of the activities, so $x_1, x_{2/}..., x_n$ are called the decision variables. As summarized in table 2.1 the values of c_j, b_i, and a_{ij} (for i=1,2,...m, and j=1,2,3,...,n)

are the input constants for the model, the C_i, b_i, and aij are also referred to as the parmateters of the model.

Notice the correspond dence between (table 2.1) .

2.2   A standard form of the model

We can now formulate the mathematical model for this general problem of allocating resources to activities. In particular, this model is to select the values for $x_1, x_2, x_3, ..., x_n$ so as to

Maximize $z = c_1 x_1 + c_2 x_2 + ... + c_n x_n$

Subject to

$a_{11}x_1 + a_{12}x_2 + ... + a_{1n} x_n \leq b_1$

$a_{21} x_1 + a_{22} x_2 + ... + a_{2n} x_n \leq b_2$

.

$a_{m1}x_1 + a_{m2} x_2 + ... + a_{mn} x_n \leq b_{mn}$

Table *2.1*

Data needed for linear programming model involving allocation of resources to activities.

Resources	Resource usage unit of Activity			Amount of Resources
	Activity			
1-	a_{11}	a_{12}	a_{1n}	b_1
2-	a_{12}	a_{22}	a_{2n}	b_2
.	…	…	…	
.	…	…	…	
.	…	…	…	
m	a_{m1}	a_{m2}	a_{mn}	b_m
Contribution to z Per unit of activity	c_1	c_2	c_n	

2.3 Other forms

We now hasten to add that the preceding model does not actually fit the natural form of some linear programming problems. The other legitimate forms are the following:-

1.minizing rather than maximizing the objective function:

Minimize $z = c_1 x_2 + c_2 x_2 + ... + c_n x_n$

2.some functional constraints with a greater than -or- equal - to inequality

$a_{i1}x_1 + a_2 x_2 + ... + a_{in}x_n \leq b_j$

For some values of i.

3.some functional constraints in equation from:-$a_{11}x_1 + a_{12}x_2 + ... + a_{in}x_n = b_j$

For some values of i.

4.Deleting the nonnegative constrains for some decision variables : x_j unrestricted in sign for some values of i.

Any problem that mixes some of all these forms with the remaining parts of the preceding model is still a linear programming problem. Our interpretation of the words allocating limited resources among competing activities may no longer apply very well. If at all: but regardless of the interpretation or context. All that is required is that the mathematical statement of the problem f it the allowable forms.

2.4 Terminology for solution of the model

A feasible solution is a solution for which all the constraint are satisfied and an **infeasible solution** is a solution for which at least one constraint is violated.

The feasible region is the collection of all feasible solutions

Notice:-

It is possible for a problem to have no feasible solutions **An optimal solution** is a feasible solution that has the most favorable value of the objective function.

The most favorable value is the largest value if the objective function is to be maximized.

Where as it is the smallest value if the objective function is to be minimized.

Most problems will have just one optimal solution However it is possible to have more than one.

Another possibility is that a problem has no optimal solutions. This occurs only if (1) it has no feasible solution or (2) the constraints do not prevent improving the value of the objective function (z) indfinitely in the favorable direction (positive or negative). The letter case is referred to as having an unbounded z.

A corner _ point feasible (CPF) solution is a solution that lies at a corner of the feasible region.

2.5 Relationship Between Optimal Solution and CPF Solution:-

Consider any linear programming problem with feasible solution and a bounded feasible region. The problem must Posses CPF solutions and at least one optimal solution. Furthermore , the best CPF solution must be an optimal solution, thus , if a problem has exactly one optimal solution it must be a CPF solution. If the problem has multiple optimal solutions, at least two must be CPF solutions.

2.6 simplex method

1- Initialization Initial non-basic variables (set equal to zero) and the slack variables to be the initial basic variables,

2- optimality test:- the current BF solution is optimal if and only if every coefficient in row is non-negative (> 0).

If it is stop; otherwise, go to an iteration to obtain the next BF solution, which involves changing one non-basic variable to basic variable (step 1) and vice verse (step 2) and then solving for the new solution (step3).

3- Iteration : step (1):- Determine the entering basic variable by selecting the variable (automatically a non-basic variable) with the negative coefficient having the largest absolute value(i.e, the "most negative" coefficient) in Eq.(O)

Put a box around the column below this coefficient and call this the point column.

Step(2):- determine the leaving basic variable by applying the minimum ratio test.

Minimum ratio Test

1- Pick out each coefficient in to pivot column that is strictly positive (> 0).

2- Divide each of these coefficients into the right side entry for the same row.

3- Identify the row that has the smallest of these ratios.

4- The basic variable for that row is the leaving basic variable, so replace that variable by the entering basic variable in the basic variable column of the next simplex tableau.

Put a box around this row and call it the pivot row. Also call the number that is in both boxes the pivot number.

Step (3) solve for the new BP solution by using elementary row operation (multiply or divide a row by a non-zero constant, and or subrat a multiple of one row to another row) to construct a new simplex table au in proper form from Gaussian elimination below the current one , and then return to the optimally test. The specific elementary row operations that need to performed are listed below.

Divide the pivot row by the pivot number, use this new pivot row in step2 and 3.

For each other row (including row 0)

That has a negative coefficient in the pivot column , add to this row the product of the absolute value of this coefficient and the new pivot row.

For each other row that has a positive coefficient in the

pivot column , subtract from this row the product of this coefficient

and the new pivot row.

2.7 Big M-method

We have presented the details of the Simplex method under the assumption that the problem is in our standard from (maximize Z subject to functional constraint in<from and non negativity constraints an all variables)and that bi ≥ 0 for all i=1,2,…,m .In this section we point .Out how to make the adjustments required for other legitimate forms of the linear programming model .You will see that all these adjustments can be made during the initialization ,so the rest of the Simplex method can ,then be applied just as you have learned it already.

The only serious problem introduced by the other forms for functional constraints

(the =or ≥ forms ,or having a negative right-hand side)lies in identifying an initial BF Solution .Before ,this initial solution was found very conveniently by letting the slack variables be the initial basic variables, so that each one just equals the nonnegative right-hand side of its equation.

Now ,something else must be done .the standard approach that is used for all these

Cases is the artificial-variable technique .This technique constructs a more

Convenient artificial problem by introducing adunmy variable called an artificial variable) into each constraint that needs one ,this new variable is introduced just for the purpose of being the initial basic variable for that question the usual non-negativity constraints are placed on these variable, and the objective function also is modified to impose an exorbitant penalty on their having values larger than zero. The iteration of the Simplex method then automatically force the artificial variable to disapper (became zero). However, rather than making this substitution and there by increasing the number of constraints it is more convenient to use the artificial-variable technique.

Chapter Three

Design of Radiation Therapy Problem

3.1 Design of Radiation Therapy Problem

A patient has been diagnosed as having a cancer at an advanced stage. The patient has a large malignant tumor in the bladder area. A patient is to receive the most advanced medical care available to give the patient every possible chance for survival. This care will involves extensive radiation therapy.

Radiation therapy problem includes using an external beam treatment machine to pass ionizing radiation through the patient's body, damaging both cancerous and healthy tissues. Several beams are administered precisely from different angles in a two-dimensional plane. Each beam delivers more radiation to the tissue near the entry point than to the tissue near the exit point. Also, scatter causes some delivery of radiation to tissue outside the direct path of the beam. Since tumor cells are typically microscopically interspersed among healthy cells, the radiation dosage throughout the tumor region must be large enough to kill the malignant cells, which are slightly more radiosensitive, yet small enough to spare the healthy cells. At the same time, in order to prevent complications that can be more serious than the disease itself, the aggregate tolerance levels. For the same reason, the total dose must be minimized to the entire healthy anatomy.

The design of radiation therapy is a very delicate process, because of the need to carefully balance all these factors. The aim of the design is to select the combination of beams to be used and the intensity of each one to generate the best possible dose distribution. At any point in the body, the dose strength is measured in units called kilorads. Once the design of treatment has been developed, it is administered in many installments, spread over several weeks.

In patient's case, the size and location of tumor make the design of treatment are even more delicate process than usual. Figure 3.1 shows a diagram of the tumor cross section viewed from above, as well as nearby critical tissues to avoid. These tissues involve critical organs (e.g., the rectum) as well as bony structures (e.g., the femurs and pelvis) that will attenuate the radiation. In this case, also shown are the entry point and direction for the only two beams that can be used with any modicum of safety.

Figure 3.1

Cross section of patient's tumor viewed from above, nearby critical tissues and the radiation beams being used.
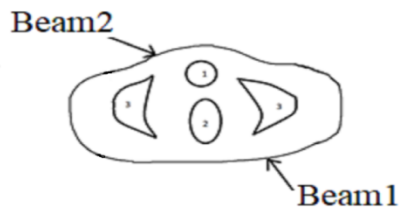
1. Bladder and tumor
2. Rectum, coccyx, etc.
3. Femur, part of pelvis, etc.

The analysis of what the resulting radiation absorption by various parts of the body would be requires a complicated process, for any proposed beam of given intensity. In abbreviation, based on careful anatomical analysis, the energy distribution within the two dimensional cross section of the tissue can be plotted on an isodose map, where the contour lines represent the dose strength as a percentage of the dose strength at the entry point. A fine grid then is placed over the isodose map. By summing the radiation absorbed in the squares containing each type of tissue, the average dose that is absorbed by the tumor, healthy anatomy, and critical tissues can be calculated. With more than one beam (administered sequentially), the radiation absorption is additive.

After analysis of this type, the medical team has carefully estimated the data needed to design patient's treatment, as summarized in table 3.1. The first column includes the areas of the body that must be considered, and then the next two columns give the fraction of the radiation dose at the entry point for each beam that is absorbed by the respective areas on average. For example, if the dose level at the entry point for beam 1 is 1 kilorad, then an average of 0.4 kilorad will be absorbed by the entire healthy anatomy in the two-dimensional plane, an average of 0.3 kilorad will be absorbed by nearby critical tissues, an average of 0.5 kilorad will be absorbed by the various parts of the tumor, and 0.6 kilorad will be absorbed by the center of the tumor. The last column gives the restrictions on the total dosage from both beams that is absorbed on average by the respective areas of the body. In particular, the average dosage absorption

for the healthy anatomy must be as small as possible, the critical tissues must not exceed 2.7 kilorads, the average over the entire tumor must equal 6 kilorads.

Table 3.1 Data for the design of patient's radiation therapy

Area	Fraction of Entry Dose Absorbed by Area (Average)		Restriction on Total Average Dosage, Kilorads
	Beam1	Beam2	
Healthy anatomy	0.4	0.5	Minimize
Critical tissues	0.3	0.1	≤ 2.7
Tumor region	0.5	0.5	$= 6$
Center of tumor	0.6	0.4	≥ 6

3.2 Formulation as a Linear Programming Model

The dosages of radiation at the two entry points are the decisions that need to be made. Hence, the two decision variables x_1 and x_2 represent the dose in kilorads at the entry point for beam 1 and beam 2, respectively. Because the total dosage reaching the healthy anatomy is to be minimized, let Z denote this quantity. The data from Table 3.1 can be used directly to formulate the linear programming model as follows :

Minimize $Z = 0.4x_1 + 0.5x_2$

Subject to

$0.3x_1 + 0.1x_2 \leq 2.7$
$0.5x_1 + 0.5x_2 = 6$
$0.6x_1 + 0.4x_2 \geq 6$
$x_1 \geq 0, x_2 \geq 0.$

This model can be solved by the Big M method. First we must describe how to deal with the third constraint.

Our approach includes introducing both a surplus variable x_5 (defined as $x_5 = 0.6x_1 + 0.4x_2 - 6$) and an artificial variable $\bar{x}_6$ as shown next .

$0.6x_1 + 0.4x_2 - x_5 = 6 \quad (x_5 \geq 0)$
$0.6x_1 + 0.4x_2 - x_5 + \bar{x}_6 = 6 \quad (x_5 \geq 0, \bar{x}_6 \geq 0).$

x_5 is called a surplus variable because its subtracts the surplus of the left-hand side over the right-hand side to convert the inequality constraint to an equivalent equality constraint. After this conversion is accomplished, the artificial variable is introduced just as for any equality constraint.

After a slack variable x_3 is introduced into the first constraint, an artificial variable $\bar{x}_4$ is introduced into the second constraint, and the Big M method is applied, so the complete artificial problem (in augmented form) is

Minimize $Z = 0.4x_1 + 0.5x_2 + M\bar{x}_4 + M\bar{x}_6$

subject to $\quad 0.3x_1 + 0.1x_2 + x_3 = 2.7$

$\quad\quad\quad\quad 0.5x_1 + 0.5x_2 + \bar{x}_4 = 6$

$\quad\quad\quad\quad 0.6x_1 + 0.4x_2 - x_5 + \bar{x}_6 = 6$

and $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $\bar{x}_4 \geq 0$, $x_5 \geq 0$, $\bar{x}_6 \geq 0$.

Note that the coefficients of the artificial variables in the objective function are $+M$, instead of $-M$, because we now are minimizing Z. Thus, even though $\bar{x}_4 > 0$ and/or $\bar{x}_6 > 0$ is possible for a feasible solution for the artificial problem, the huge unit penalty of $+M$ prevents this from occurring in an optimal solution.

A straightforward way of minimizing Z with the Simplex method is to exchange the roles of the positive and negative coefficients in row 0 for both the optimality test and step 1 of an iteration. However, rather than changing our instructions for the Simplex method for this case, we present the following simple way of converting any minimization problem to an equivalent maximization problem:

Minimizing $Z = \sum_{j=1}^{n} c_j x_j$

is equivalent to

maximizing $\quad -Z = \sum_{j=1}^{n} (-c_j) x_j$

i.e., the two formulations yield the same optimal solution (s).

The two formulations are equivalent because the smaller Z is, the larger $-Z$ is, so the solution that gives the smallest value of Z in the entire feasible region must also give the largest value of $-Z$ in this region.

Therefore, in the radiation therapy problem, we make the following change in the formulation:

Minimize $Z = 0.4x_1 + 0.5x_2$

Maximize $-Z = -0.4x_1 - 0.5x_2$.

After artificial variables $\bar{x}_4$ and $\bar{x}_6$ are introduced and then the Big M method is applied, the corresponding conversion is

Minimize $Z = 0.4x_1 + 0.5x_2 + M\bar{x}_4 + M\bar{x}_6$

Maximize $-Z = -0.4x_1 - 0.5x_2 - M\bar{x}_4 - M\bar{x}_6$.

Now, we are nearly to apply the Simplex method to the radiation therapy problem. By using the maximization form, the entire system of equations is now

(0) $-Z+0.4x_1+0.5x_2+M\bar{x}_4+M\bar{x}_6 = 0$

(1) $0.3x_1+0.1x_2+x_3=2.7$

(2) $0.5x_1+0.5x_2+\bar{x}_4=6$

(3) $0.6x_1+0.4x_2-x_5+\bar{x}_6=6.$

The basic variables $(x_3, \bar{x}_4, \bar{x}_6)$ for the initial basic feasible solution (for this artificial problem) are shown in bold type.

Note that this system of equations is not yet in proper form from Gaussian elimination, as required by the simplex method, since the basic variables $\bar{x}_4$ and $\bar{x}_6$ still need to be algebraically eliminated from Eq. (0). Because $\bar{x}_4$ and $\bar{x}_6$ both have a coefficient of M, Eq. (0) needs to have subtracted from it both M times Eq. (2) and M times Eq. (3). The calculations for all the coefficients (and the right-hand sides) are summarized below, where the vectors are the relevant rows of the simplex tableau corresponding to the above system of equations.

Row 0:

[0.4,	0.5,	0,	M,	0,	M,	0]
$-M[0.5,$	0.5,	0,	1,	0,	0,	6]
$-M[0.6,$	0.4,	0,	0,	$-1,$	1,	6]

New row $0=[-1.1M+0.4,$ $-0.9M+0.5,$ 0, 0, M, 0, $-12M]$

The resulting initial simplex tableau, ready to begin the simplex method, is shown at the top of Table 4.1. Applying the simplex method in just the usual way then yields the sequence of simplex tableaux shown in the rest of Table 4.1.

Table 3.1 The Big M method for the radiation therapy problem

Basic Variable	Eq.	Z	Coefficient of: x_1	x_2	x_3	$\bar{x}_4$	x_5	$\bar{x}_6$	Right Side
Z	(0)	-1	$-1.1M + 0.4$	$-0.9M + 0.5$	0	0	M	0	$-12M$
x_3	(1)	0	0.3	0.1	1	0	0	0	2.7
$\bar{x}_4$	(2)	0	0.5	0.5	0	1	0	0	6
$\bar{x}_6$	(3)	0	0.6	0.4	0	0	-1	1	6
Z	(0)	-1	0	$-\frac{16}{30}M + \frac{11}{30}$	$\frac{11}{3}M - \frac{4}{3}$	0	M	0	$-2.1M - 3.6$
x_1	(1)	0	1	$\frac{1}{3}$	$\frac{10}{3}$	0	0	0	9
$\bar{x}_4$	(2)	0	0	$\frac{1}{3}$	$-\frac{5}{3}$	1	0	0	1.5
$\bar{x}_6$	(3)	0	0	0.2	-2	0	-1	1	0.6
Z	(0)	-1	0	0	$-\frac{5}{3}M + \frac{7}{3}$	0	$-\frac{5}{3}M + \frac{11}{6}$	$\frac{8}{3}M - \frac{11}{6}$	$-0.5M - 4.7$
x_1	(1)	0	1	0	$\frac{20}{3}$	0	$\frac{5}{3}$	$-\frac{5}{3}$	8
$\bar{x}_4$	(2)	0	0	0	$\frac{5}{3}$	1	$\frac{5}{3}$	$-\frac{5}{3}$	0.5
x_2	(3)	0	0	1	-10	0	-5	5	3
Z	(0)	-1	0	0	0.5	$M-1.1$	0	M	-5.25
x_1	(1)	0	1	0	5	-1	0	0	7.5
x_5	(2)	0	0	0	1	0.6	1	-1	0.3
x_2	(3)	0	0	1	-5	3	0	0	4.5

Hence, the optimal design is to use a total dose at the entry point of 7.5 kilorads for beam 1 and 4.5 kilorads for beam2.

3.3 Conclusions

Mathematical modeling is a powerful tool for planning and decision making in healthcare. In this paper, the linear programming model for the radiation therapy problem is presented and this model is solved by using Big M method to obtain the optimal design.

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الخلاصة

في هذا البحث نقدم اساليب لدمج الطرق والمهارات في الرياضيات والحسابات مع المعرفة المتخصصة لخبراء الرعاية الصحية للوصول الى حلول ملائمة للمشاكل في قطاع الرعاية الصحية. في هذا البحث تم تصميم نموذج رياضي لمسألة العلاج بالاشعاع بأستخدام البرمجة الخطية. تم حل هذا النموذج بأستخدام طريقة Big M Method

كلمات مفتاحية : النموذج الرياضي, البرمجة الخطية, العلاج الاشعاعي, الرعاية الصحية.

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مشروع تم تقديمه الى قسم علوم الرياضيات بجامعة ديالى في انجاز جزئي لدرجة البكالوريوس في الرياضيات

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